WHAT IS OLD, WHAT IS NEW, AND WHAT TO DO?

Tuvi Etzion

Computer Science Department

TECHNION
Israel Institute of Technology

Dubrovnik, Croatia, April 8, 2016

To Honor the Memory

Rudolph Ahlswede 1938 -- 2010

Ralf Kötter 1963 -- 2009

And Our COST Friend

Axel Kohnert 1962 -- 2013

Outline

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Before we knew on network coding

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Before we knew on network coding

From network codes to subspace codes

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Before we knew on network coding

From network codes to subspace codes

Codes and designs over vector spaces

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Before we knew on network coding

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Four years in the COST Action

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Before we knew on network coding

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Open problems and future research

Before Network Coding

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The father of Projective Geometry

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Girard Desargues 1593 -- 1661

Before Network Coding

The father of Projective Geometry

Girard Desargues 1593 -- 1661

Grandfather?

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Pappus 300 AD

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Who else?

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Blaise Pascal 1623 -- 1662

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What next?

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What next?

unfortuntely,

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Pappus 300 AD

Who else?

Blaise Pascal 1623 -- 1662

What next?

unfortuntely,

a clinical death

Projective Geometry Reborn

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Hermann Grassmann 1809 – 1877

Projective Geometry Reborn

The Grassmann graph

Hermann Grassmann 1809 -- 1877

The Great Geometers

The Great Geometers

Julius Plücker 1801 -- 1868

The Great Geometers

Plücker coordinates

Julius Plücker 1801 -- 1868

The Great Geometers

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Gino Fano 1871 -- 1952

The Great Geometers

Fano plane

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The Great Geometers

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Felix Klein 1849 -- 1925

The Great Geometers

Klein quartic

Felix Klein 1849 – 1925

And Block Designers

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Jakob Steiner 1796 – 1863

And Block Designers

Steiner systems

Jakob Steiner 1796 -- 1863

And The Great

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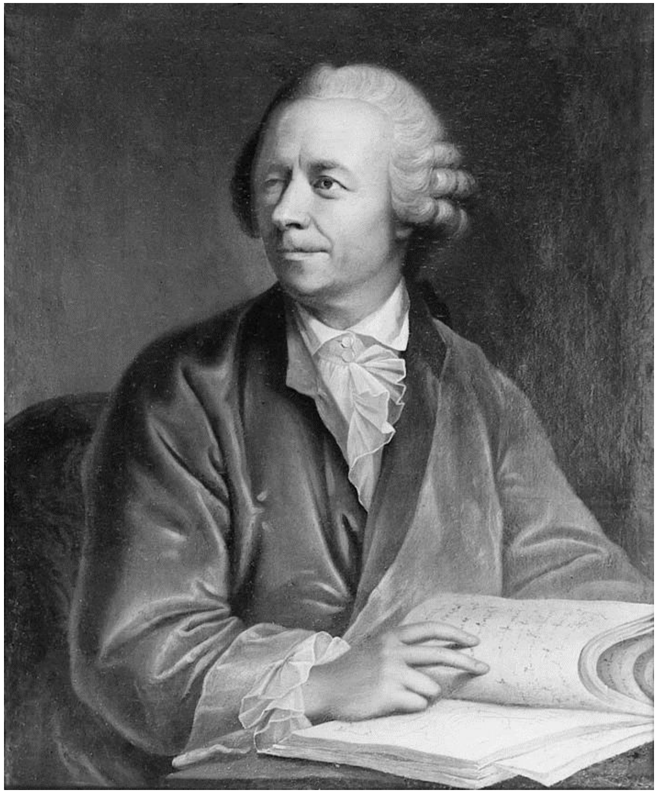

Leonhard Euler 1707 -- 1783

q-Analogs

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Jacques Tits 1930 -

1957

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Peter Cameron 1947 -

1974

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Philippe Delsarte 1942 -

1976

t-Designs Over Vector Spaces

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1987, 1996

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D. K. Ray-Chaudhuri, N. M. Singhi

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M. Miyakawa, A. Munemasa, S. Yoshiara 1995

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M. Miyakawa, A. Munemasa, S. Yoshiara — 1995

M. Braun, A. Kerber, R. Laue — 2005

Codes, Designs in Vector Spaces

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1987

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W. J. Martin, X. J. Zhu 1995

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M. Schwartz, T. E. 2002

Results in Projective geometry

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Partial spread

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A set of disjoint k-subspaces.

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A set of disjoint k-subspaces.

Blocking set

Results in Projective geometry

Partial spread

A set of disjoint k-subspaces.

Blocking set

A set of k-subspaces which are incident to each t-subspace, $t > k$.

Results in Projective geometry

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Albrecht Beutelspacher 1950 -

Results in Projective geometry

The size of a partial spreads

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The size of a partial spreads

1-parallelisms - partitions of all 1-subspaces into spreads.

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The size of a partial spreads

1-parallelisms - partitions of all 1-subspaces into spreads.

Geometric spreads - optimal q-covering designs.

Hamming and Preparata Codes

Hamming and Preparata Codes

The codewords of weight three in the Hamming code correspond to 1-dimensional subspaces. The union of words of weight three in certain translates of the Preparata code consists of exactly these codewords. These words in each such translate corresponds to a 1-spread. Thus, we have a 1-parallelism for the 1-dimensional subspaces.

Rank-Metric Codes

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Rank-metric codes played an important role in error-correcting codes for network codes and in error-correcting codes for network coding. Comprehensive work, upper bounds on their size and constructions which attain these bounds were found.

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P. Delsarte 1978, E. M. Gabidulin 1985, R. M. Roth 1991

Routing

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min-cut/max-flow Theorem

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K. Menger 1927

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Edmonds Theorem

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In a directed graph $G = (V, E)$ there are k edge disjoint spanning trees rooted at $r \in V$ if and only if $k \leq C_G(r, V \setminus \{r\})$.

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Maximaizing the multicast rate is an NP-hard problem with reduction to the Steiner tree problem.

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J. Edmonds 1972

Maximaizing the multicast rate is an NP-hard problem with reduction to the Steiner tree problem.

K. Jain, M. Mahdian, M. R. Salavatipour 2003

Network Codes to Subspace Codes

Network Codes to Subspace Codes

Multicast Network

Network Codes to Subspace Codes

Multicast Network

A multicast network is a directed acyclic graph containing a single source node and a collection of N destination nodes. The source node has a set of h messages from a fixed alphabet and each destination node tries to recover all the messages.

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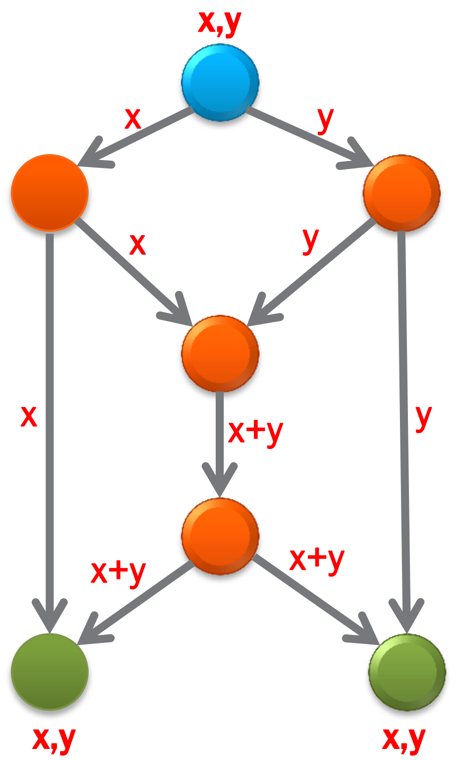
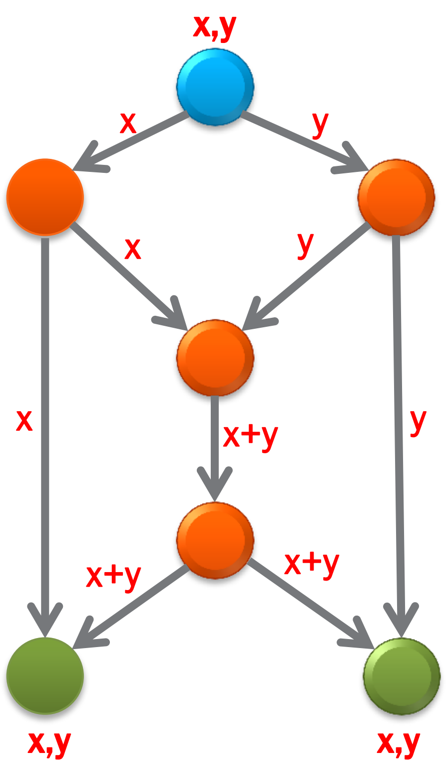


Network Codes to Subspace Codes

The butterfly network



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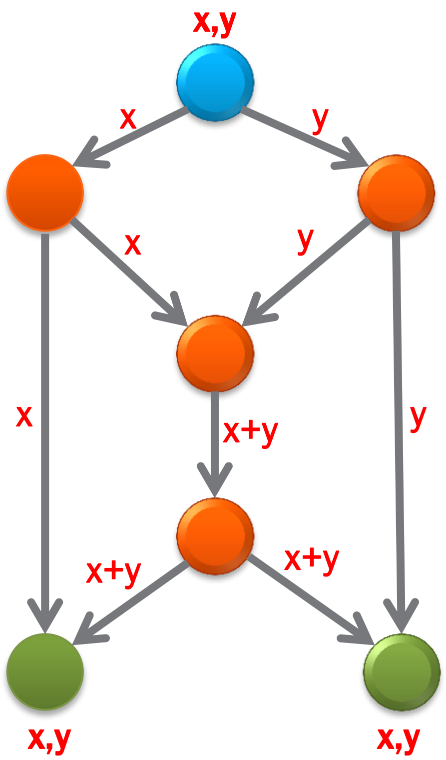
$\mathbf{x}, \mathbf{y} \in \{\mathbf{0}, \mathbf{1}, \dots, \boldsymbol{n} - \mathbf{1}\}$



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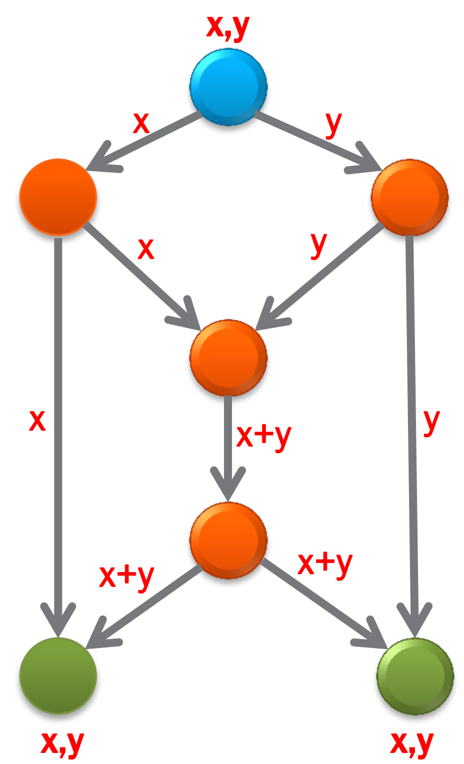

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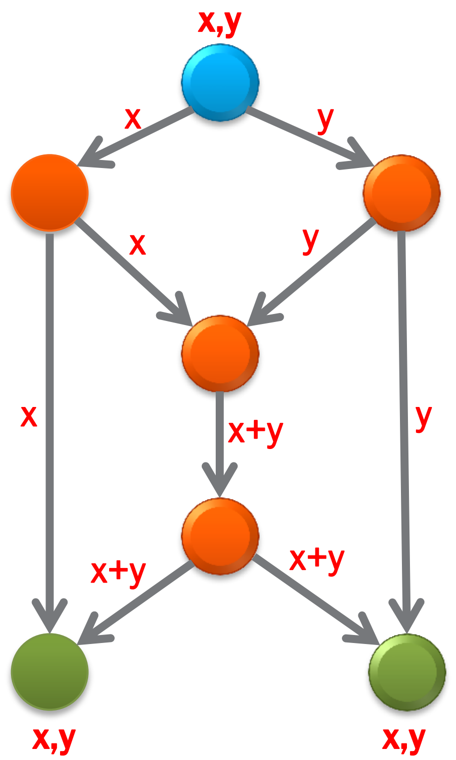

R. Ahlswede, N. Cai, S.-Y. Li, R. W. Yeung 2000

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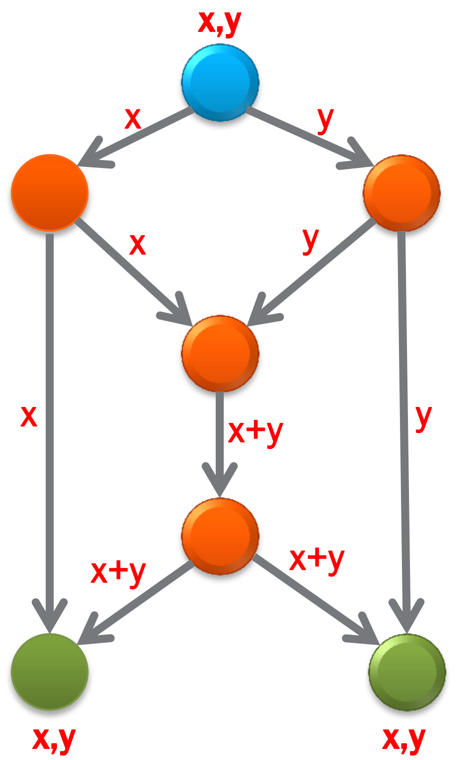

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S.-Y. Li, R. W. Yeung, N. Cai 2003

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Min-Cut/Max-Flow Theorem for Multicast Networks

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Network Codes to Subspace Codes

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A multicast network is solvable if the min-cut to each destination is h.

Network Codes to Subspace Codes

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R. Kötter, M. Médard 2003

Network Codes to Subspace Codes

Algebraic Approach for Network Coding

R. Kötter, M. Médard 2003

Network Codes to Subspace Codes

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Polynomial Time Algorithm for Solvable Multicast Networks

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A polynomial time algorithm to find the network code of a solvable multicast network with N receivers. The solution is over any field $\mathbb{F}_q$ such that $q \geq N$.

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Polynomial Time Algorithm for Solvable Multicast Networks

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S. Jaggi, P. Sanders, P. A. Chou, M. Effros, S. Egner, K. Jain, L. M. G. M. Tolhuizen 2005

Network Codes to Subspace Codes

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Random Network Coding

Network Codes to Subspace Codes

Random Network Coding

Network nodes independently and randomly select linear mappings from inputs links onto outputs links over some field.

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T. Ho, M. Medard, R. Kötter, D. R. Karger, M. Effros, J. Shi, B. Leong 2006

Network Codes to Subspace Codes

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R. Kötter, and F. Kschischang, 2008

Network Codes to Subspace Codes

The Operator Channel

R. Kötter, and F. Kschischang, 2008

Network Codes to Subspace Codes

The Operator Channel

Subspace codes for error-correction in random network coding.

R. Kötter, and F. Kschischang, 2008

Network Codes to Subspace Codes

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Lifted Rank-Metric Codes

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Large constant dimension codes can be constructed by lifting rank-metric codes, especially maximum rank distance (MRD) codes.

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D. Silva, F. Kschischang, R. Kötter 2008

Network Codes to Subspace Codes

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Metrics for Error-Correcting Network Codes

Network Codes to Subspace Codes

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Which metric to use: rank distance, subspace distance, or injection distance.

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D. Silva, F. Kschischang 2009

Codes and Designs Over GF(q)

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Error-correcting codes in the projective space

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Basic bounds, linear programming, cyclic codes, perfect codes.

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T. E. and A. Vardy 2008, 2011

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Covering Designs in the Grassmann space

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Basic bounds and constructions for covering designs over GF(q).

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T. E. and A. Vardy 2011

Codes and Designs Over GF(q)

Subspace Codes via Ferrers Diagram and Rank-Metric Codes

Codes and Designs Over GF(q)

Subspace Codes via Ferrers Diagram and Rank-Metric Codes

Representation of subspaces, Ferrers diagram rank-metric codes, punctured codes.

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Codes based on Lifted MRD Codes

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Properties of Grassmannian codes which are defined as orbits of a subgroup of the general linear group.

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A.-L. Trautmann, F. Manganiello, M. Braun, J. Rosenthal 2013

Codes and Designs Over GF(q)

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Covering Codes

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New constructions (upper bounds), mainly ones based on lifted MRD codes.

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T. E. 2014

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$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{(q^n-1)(q^{n-1}-1)\cdots(q^{n-k+1}-1)}{(q^k-1)(q^{k-1}-1)\cdots(q-1)}$$

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$$|G_q(n,k)| = \begin{bmatrix} n \\ k \end{bmatrix}_q$$

q-Steiner Systems

q-Steiner Systems

A q-Steiner system $S(t,k,n)_q$ is a pair (N,B), where N is an n-dimensional space over $\mathbb{F}_q$ and B is set of k-dimensional subspaces (called blocks) of N such that each t-dimensional subspace of N is contained in exactly one block of B.

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$$\left|S(t,k,n)_q\right| = {\begin{bmatrix} n \\ t \end{bmatrix}_q} \Big/ {\begin{bmatrix} k \\ t \end{bmatrix}_q}$$

Codes and Designs Over GF(q)

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Algebraic, Combinatorics, and Applications, Thurnau, Germany 2010

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Castle Meeting on Coding Theory and Applications, Cardona, Spain 2011

Asymptotic Behavior

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$A(n) \sim B(n)$ if $\lim {}^{A(n)}/_{B(n)} = 1$ as $n \to \infty$.

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Theorem

If q, k, and t are fixed integers with $0 \leq t \leq k$, q a prime power, then the size $P(t,k,n)$ (the largest size of a set with k–subspaces (blocks) of an n-space N such that each t-subspace of N appears in exactly one block) satisfies

$$P(t,k,n) \sim {}^{\begin{bmatrix} n \\ t \end{bmatrix}_q}\Big/_{\begin{bmatrix} k \\ t \end{bmatrix}_q}$$

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S.Blackburn, T. E. 2012

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S.Blackburn, T. E. 2012

Same result for covering.

Spreads

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α^r is primitive in the subfield $GF(q^k)$ of $GF(q^n)$

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α^r is primitive in the subfield $GF(q^k)$ of $GF(q^n)$

$\{0, \alpha^i, \alpha^{i+r}, \alpha^{i+2r}, \ldots, \alpha^{i+(2^k-2)r}\}, 0 \le i \le r-1,$ are closed under addition in $GF(q^n)$ $\Rightarrow$ subspaces $\Rightarrow$ $S(1,k,n)_q$

Four Years of the COST Action

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$S(2, 3, 13)_2$

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$S(2, 3, 13)_2$

Ascona 2012

Four Years of the COST Action

$S(2,3,13)_2$

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M. Braun, T. E., P. Östergård, A. Vardy, A. Wassermann, 2013

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Bergen 2013

M. Braun, T. E., P. Östergård, A. Vardy, A. Wassermann, 2013

Four Years of the COST Action

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$\boldsymbol{S(2,3,13)_2}$

Four Years of the COST Action

$S(2,3,13)_2$

α primitive in $GF(2^{13})$

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$V = \{0, \alpha^{i_1}, \alpha^{i_2}, \alpha^{i_3}, \alpha^{i_4}, \alpha^{i_5}, \alpha^{i_6}, \alpha^{i_7}\}$

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Frobenius map

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normalizer of Singer subgroup automoprphism

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15 representatives

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15 representatives

1 597 245
3–dimensional subspaces

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Designs Over Vector Spaces

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Designs Over Vector Spaces

Ghent 2013

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Designs Over Vector Spaces

Ghent 2013

Derived and residual subspace designs.

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M. Kiermaier and R. Laue 2015

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Fano Plane

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Fano Plane

Ghent 2013

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If a 2-Fano plane exists it does not have a large automorphism group.

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M. Braun, M. Kiermaier, N. Nakić 2015

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Istanbul 2015

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M. Braun, M. Kiermaier, N. Nakić 2015

The structure of the q-Fano plane if exists.

Istanbul 2015

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Ghent 2013

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M. Braun, M. Kiermaier, N. Nakić 2015

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Istanbul 2015

T. E. 2015

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Rank-Metric Codes

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Bordeaux 2014

Four Years of the COST Action

Rank-Metric Codes

Bordeux 2014

Palmela 2014

Four Years of the COST Action

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Bordeux 2014

Palmela 2014

Constructions of Ferrers diagram rank-metric codes and a related Anticode bound.

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T. E., E. Gorla, A. Ravagnani, A. Wachter-Zeh 2014

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New Codes

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New Codes

Bordeux 2014

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Bordeux 2014

ALCOMA 2015

Four Years of the COST Action

New Codes

Bordeaux 2014

ALCOMA 2015

Large constant dimension codes of length 6 and length 8 over any given finite field.

Four Years of the COST Action

New Codes

Bordeaux 2014 | **ALCOMA 2015**

Large constant dimension codes of length 6 and length 8 over any given finite field.

A. Cossidente, F. Pavese 2014
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Subspace Polynomial and Cyclic Subspace Codes

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Palmela 2014

Four Years of the COST Action

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Subspace polynomial, cyclic mapping, Frobenius mapping, relatively large cyclic subspace codes.

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Istanbul 2015

K. Otal and F. Özbudak 2015

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Equidistant Codes

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Equidistant Codes

Bordeaux 2014

Four Years of the COST Action

Equidistant Codes

Theory for equidistant Grassmannian codes and rank-metric codes

Bordeux 2014

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R.D. Barrolleta, D. Bartoli, M. De Boeck, E. Suárez Canedo, L. Storme, A.-E. Riet, P. Vandendriessche

Istanbul 2015

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Vector Network Coding Outperforms Scalar Network Coding

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Vector Network Coding Outperforms Scalar Network Coding

Multicast network which is solved with vectors of length t over $\mathbb{F}_q$ and can be solved with scalar linear network coding only over a field of order $q^{(\ell-1)t^2/\ell}$, where 2ℓ is the number of messages.

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T. E. and A. Wachter-Zeh 2016

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T. E. and A. Wachter-Zeh 2016

Constructions and bounds are based on rank-metric codes and subspace codes.

Open Problems, Future Reseach

Open Problems, Future Reseach

Bounds on the Alphabet size for a given number h of messages and N receivers.

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Does there exist a multicast network with two messages in which vector network coding outperforms scalar network coding?

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Bounds on the Alphabet size for a given number h of messages and N receivers.

Does there exist a multicast network with two messages in which vector network coding outperforms scalar network coding?

Is there a multicast network in which exactly h edge disjoint paths are used to each receiver, and vector network coding outperforms scalar network coding.

Open Problems, Future Reseach

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Find new q-Steiner systems.

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Find new q-Steiner systems.

Prove the nonexistence of some currently possible q-Steiner systems.

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Prove the nonexistence of some currently possible q-Steiner systems.

Does there exists q-Steiner system $S(2,3,7)_q$ (q-Fano plane).

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Does there exists q-Steiner system $S(2,3,7)_q$ (q-Fano plane).

Improve the bounds on the sizes of partial spreads.

Open Problems, Future Reseach

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Constructions for large cyclic codes.

Open Problems, Future Reseach

Constructions for large cyclic codes.

New constructions and bounds on subspaces codes which are not constant dimension codes.

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Prove that current upper bounds are asymptotically optimal for new parameters.

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Prove that current upper bounds are asymptotically optimal for new parameters.

Find new applications for subspace codes.

My Former Students, Postdocs

Moshe Schwartz, Natalia Silberstein, Netanel Raviv, Antonia Wachter-Zeh

THANK YOU