## On mixed dimension subspace codes

#### Francesco Pavese (joint work with A. Cossidente and L. Storme)

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## $(\mathcal{S}(V), d_s), (\mathcal{G}_q(n, k), d_s)$ are metric spaces

- S(V) set of all subspaces of PG(n-1, q),
- $G_q(n, k)$  set of all *k*-dimensional subspaces of PG(n-1, q), *Grassmannian*,
- $d_s(U, U') = \dim(U + U') \dim(U \cap U')$  subspace distance.

#### The main problem in subspace coding theory

- determination of the maximum size of codes with given minimum distance,
- the classification of the corresponding optimal codes.

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## Motivation

Codes in the projective space and codes in the Grassmannian over a finite field  $\longrightarrow$  error control in random linear network coding.

- R. Kötter, F. R. Kschischang, Coding for Errors and Erasures in Random Network Coding, *IEEE Trans. Inf. Theory* 54 (2008), 3579-3591.
- T. Etzion, Problems on *q*-analogs in coding theory, *preprint* (arXiv:1305.6126).

#### *q*-analogs

- subspace codes are q-analogs of constant weight codes
- subsets —> subspaces of a vector space over a finite field
- orders —> dimensions of the subspaces.

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 $d = 3, \ n = 5$ subspaces of PG(4, q)

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largest partial line spread of PG(4, q)

C optimal  $(5,3)_q$  subspace code lines contained in C are pairwise disjoint C contains at most  $q^3 + 1$  lines

Dual argument

planes contained in C are pairwise intersecting in a point C contains at most  $q^3 + 1$  planes

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## if C consists of lines and planes $|C| < 2(a^3 + 1)$

#### Properties

C contains at most one point, C contains at most one solid, if C contains one point, then C contains at most  $q^3$  planes, if C contains one solid, then C contains at most  $q^3$  lines.

 $\mathcal{A}_q(5,3) \leq 2(q^3+1)$ 

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## $\mathcal{A}_q(5,3) \leq 2(q^3+1)$

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if  $\mathcal C$  consists of lines and planes  $|\mathcal C| \leq 2(q^3+1).$ 

Properties

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if C contains one point, then C contains at most  $q^3$  planes, if C contains one solid, then C contains at most  $q^3$  lines.

$$\mathcal{A}_q(5,3) \leq 2(q^3+1)$$

# C consists of one point, q<sup>3</sup> + 1 lines and q<sup>3</sup> planes; C consists of q<sup>3</sup> lines, q<sup>3</sup> + 1 planes and one solid; C consists of one point, q<sup>3</sup> lines, q<sup>3</sup> planes and one solid C consists of q<sup>3</sup> + 1 lines and q<sup>3</sup> + 1 planes.

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I) C consists of one point, q<sup>3</sup> + 1 lines and q<sup>3</sup> planes;
II) C consists of q<sup>3</sup> lines, q<sup>3</sup> + 1 planes and one solid;
III) C consists of one point, q<sup>3</sup> lines, q<sup>3</sup> planes and one solid
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- I) C consists of one point,  $q^3 + 1$  lines and  $q^3$  planes; II) C consists of  $q^3$  lines,  $q^3 + 1$  planes and one solid;
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#### $q = p^h$ odd prime power

$$PG(4, q)$$

$$\ell : X_3 = X_4 = X_5 = 0,$$

$$\pi : X_4 = X_5 = 0,$$

$$\Sigma_i : X_4 = \omega^{i-1} X_5, 1 \le i \le q - 1,$$

$$\Sigma_q : X_4 = 0,$$

$$\Sigma_{a+1} : X_5 = 0,$$

 $\ell \subset \pi \subset \Sigma_i$ 

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- $\pi \setminus \ell$ ,
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 $a, b, c \in GF(q)$  $X^3 + aX^2 + bX + c = 0$  is irreducible over GF(q),

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ight),$  $G = \{M_{r.s.t} \mid r, s, t \in \mathrm{GF}(q)\} \leq \mathrm{PGL}(5, q),$ *p*–group of order  $q^3$ .

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A line–orbit of type e) is a partial line spread of PG(4, q).

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#### $\sigma$ plane,

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 $\mathcal{L}$  line-orbit of type e), no line of  $\mathcal{L}$  is contained in  $\sigma$ 

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 $q = p^h$  even prime power

$$\ell : X_{1} = X_{4} = X_{5} = 0,$$
  

$$\pi : X_{4} = X_{5} = 0,$$
  

$$\Sigma : X_{1} = 0,$$
  

$$\Sigma \cap \pi = \ell$$

#### pencil ${\cal F}$

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pencil  $\mathcal{F}$ 

- solid  $\Sigma$ ,

- cone C: vertex N = (1, 0, 0, 0, 0)

base  ${\mathcal H}$  hyperbolic quadric of  $\Sigma$ 

 $X_2X_5 + X_3X_4 = 0$ 

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$$G \simeq C_{q+1} imes (E_q imes C_{q-1}) \le ext{PGL}(5,q)$$
  
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Francesco Pavese On mixed dimension subspace codes

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$$\alpha \in \mathrm{GF}(q)$$

$$X^2 + X + \alpha = 0 \text{ is irreducible over } \mathrm{GF}(q),$$

$$M_{a,b,c,d} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & ac & \alpha ad & bc & \alpha bd \\ 0 & ad & a(c+d) & bd & b(c+d) \\ 0 & 0 & 0 & a^{-1}c & \alpha a^{-1}d \\ 0 & 0 & 0 & a^{-1}d & a^{-1}(c+d) \end{pmatrix},$$

 $G = \{M_{a,b,c,d} \mid a,b,c,d \in \mathrm{GF}(q), a \neq 0, c^2 + cd + \alpha d^2 = 1\}$ 

$$egin{aligned} G &\simeq C_{q+1} imes (E_q imes C_{q-1}) \leq ext{PGL}(5,q) \ & |G| = q^3 - q \ & G ext{ fixes each quadric of } \mathcal{F} \end{aligned}$$

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#### $\Sigma \setminus \mathcal{H}, \mathcal{C} \setminus (\pi \cup \mathcal{H}), \mathcal{Q}_i \setminus \mathcal{H}, 1 \leq i \leq q-1$

*r* line meeting each of  $\Sigma \setminus \mathcal{H}, C \setminus (\pi \cup \mathcal{H}), Q_i \setminus \mathcal{H},$   $1 \le i \le q - 1$  in exactly one point, *r*<sup>G</sup> set of  $q^3 - q$  lines forming partial spread

there are  $q^2 - q$  line–orbits of this type

there are  $q^2 - q$  plane–orbits of size  $q^3 - q$  consisting of planes mutually intersecting in one point.

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#### $\mathcal{R}_1$ regulus of $\mathcal{H}$ containing $\ell$ , $\mathcal{R}_2$ opposite regulus,

 $\mathcal{P}_1$  be a plane-orbit of size  $q^3 - q$ there exists a line-orbit  $\mathcal{L}$  of size  $q^3 - q$ no line of  $\mathcal{L}$  is contained in a plane of  $\mathcal{P}_1$  $\mathcal{P}_2$  set of q + 1 planes generated by a line of  $\mathcal{R}_1$  and the point N

 $\mathcal{P}_1 \cup \mathcal{P}_2$  set of  $q^3 + 1$  planes mutually intersecting in a point  $\mathcal{L} \cup \mathcal{R}_2$  set of size  $q^3 + 1$  disjoint lines

 $\mathcal{L} \cup \mathcal{R}_2 \cup \mathcal{P}_1 \cup \mathcal{P}_2$ optimal code of type /\

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 $\mathcal{R}_1$  regulus of  $\mathcal{H}$  containing  $\ell$ ,  $\mathcal{R}_2$  opposite regulus,

#### $\mathcal{P}_1$ be a plane–orbit of size $q^3 - q$

there exists a line–orbit  $\mathcal L$  of size  $q^3 - q$ no line of  $\mathcal L$  is contained in a plane of  $\mathcal P_1$  $\mathcal P_2$  set of q + 1 planes generated by a line of  $\mathcal R_1$  and the point N

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 $\mathcal{P}_1$  be a plane–orbit of size  $q^3 - q$ there exists a line–orbit  $\mathcal{L}$  of size  $q^3 - q$ 

no line of  $\mathcal{L}$  is contained in a plane of  $\mathcal{P}_1$ 

 $\mathcal{P}_2$  set of q + 1 planes generated by a line of  $\mathcal{R}_1$  and the point N

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optimal code of type IV)

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 $\mathcal{L} \cup \mathcal{R}_2 \cup \mathcal{P}_1 \cup \mathcal{P}_2$ optimal code of type *IV*)

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#### $\mathcal{A}_2(5,3)=18$

• T. Etzion, A. Vardy, Error-correcting codes in projective space, *IEEE Trans. Inform. Theory* 57 (2011), no. 2, 1165-1173.

$$\mathcal{A}_q(5,3)=2(q^3+1)$$

 T. Honold, M. Kiermaier, S. Kurz, Constructions and Bounds for Mixed-Dimension Subspace Codes, *preprint* arXiv:1512.06660.

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## THANK YOU

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